Steel Beam Moment Strength

Flexural strength of a steel wide-flange beam section.

Assumptions

[ASSUME] AISC 14th Edition controls design
[ASSUME] Beam web is unstiffened

Inputs

Beam ultimate moment demand; $M_u = 30\ \text{kip} - \text{ft}$

Beam unbraced length; $L_b = 20\ \text{ft}$

Beam section size; $\text{section} = \text{W18X40}$

Steel yield strength; $F_y = 50\ \text{ksi}$

Modulus of elasticity; $E = 29000\ \text{ksi}$

Lateral-torsional buckling modification factor; $C_b = 1$ [AISC F1(3)]

Section Properties

$S_x = 68.4\ \text{in}^3$

$Z_x = 78.4\ \text{in}^3$

$r_y = 1.27\ \text{in}$

$r_{ts} = 1.56\ \text{in}$

$J = 0.81\ \text{in}^4$

$h_o = 17.4\ \text{in}$

$b_f/2t_f = 5.73$

$h/t_w = 50.9$

1. Beam Flexural Capacity

Flexural resistance factor
$\phi_b = 0.9$ [AISC F1(1)]

1.1. Section Compactness

$$\lambda_{pf} = 0.38 \cdot \sqrt{\frac{E}{F_y}} = 0.38 \cdot \sqrt{\frac{29000\ \text{ksi}}{50\ \text{ksi}}}$$ [AISC Table B4.1b(10)]

$$\therefore \lambda_{pf} = 9.152$$

$$Check\ b_f/2t_f \ \leq \ \lambda_{pf}$$
$$5.73 \ \leq \ 9.152$$
$$\therefore CompactFlange$$

$$\lambda_{pw} = 3.76 \cdot \sqrt{\frac{E}{F_y}} = 3.76 \cdot \sqrt{\frac{29000\ \text{ksi}}{50\ \text{ksi}}}$$ [AISC Table B4.1b(15)]

$$\therefore \lambda_{pw} = 90.55$$

$$Check\ h/t_w \ \leq \ \lambda_{pw}$$
$$50.9 \ \leq \ 90.55$$
$$\therefore CompactWeb$$

1.2. Plastic Moment Strength

Nominal plastic moment strength

$$M_p = \frac{F_y \cdot Z_x}{12\ \text{in/ft}} = \frac{50\ \text{ksi} \cdot 78.4\ \text{in}^3}{12\ \text{in/ft}}$$ [AISC Eq. F2-1]

$$\therefore M_p = 326.7\ \text{kip} - \text{ft}$$

1.3. Yielding Strength

$$M_{ny} = \frac{F_y \cdot Z_x}{12\ \text{in/ft}} = \frac{50\ \text{ksi} \cdot 78.4\ \text{in}^3}{12\ \text{in/ft}}$$ [AISC Eq. F2-1]

$$\therefore M_{ny} = 326.7\ \text{kip} - \text{ft}$$

1.4. Lateral-Torsional Buckling

$$L_p = \frac{1.76 \cdot r_y \cdot \sqrt{\frac{E}{F_y}}}{12\ \text{in/ft}}$$

$$= \frac{1.76 \cdot 1.27\ \text{in} \cdot \sqrt{\frac{29000\ \text{ksi}}{50\ \text{ksi}}}}{12\ \text{in/ft}}$$ [AISC Eq. F2-5]

$$\therefore L_p = 4.486\ \text{ft}$$

$$c = 1$$ [AISC Eq. F2-8a]

$$L_r = \frac{\frac{1.95 \cdot r_{ts}}{12\ \text{in/ft}} \cdot E}{0.7 \cdot F_y} \cdot \sqrt{\frac{J \cdot c}{S_x \cdot h_o} + \sqrt{\left(\frac{J \cdot c}{S_x \cdot h_o}\right)^2 + 6.76 \cdot \left(\frac{0.7 \cdot F_y}{E}\right)^2}}$$

$$= \frac{\frac{1.95 \cdot 1.56\ \text{in}}{12\ \text{in/ft}} \cdot 29000\ \text{ksi}}{0.7 \cdot 50\ \text{ksi}} \cdot \sqrt{\frac{0.81\ \text{in}^4 \cdot 1}{68.4\ \text{in}^3 \cdot 17.4\ \text{in}} + \sqrt{\left(\frac{0.81\ \text{in}^4 \cdot 1}{68.4\ \text{in}^3 \cdot 17.4\ \text{in}}\right)^2 + 6.76 \cdot \left(\frac{0.7 \cdot 50\ \text{ksi}}{29000\ \text{ksi}}\right)^2}}$$ [AISC Eq. F2-6]

$$\therefore L_r = 13.1\ \text{ft}$$

$$\rightarrow L_b > L_r$$

$$F_{cr} = \frac{C_b \cdot (\pi)^2 \cdot E}{\left(\frac{L_b \cdot 12\ \text{in/ft}}{r_{ts}}\right)^2} + \sqrt{1 + \frac{0.078 \cdot J \cdot c}{S_x \cdot h_o} \cdot \left(\frac{L_b \cdot 12\ \text{in/ft}}{r_{ts}}\right)^2}$$

$$= \frac{1 \cdot (3.142)^2 \cdot 29000\ \text{ksi}}{\left(\frac{20\ \text{ft} \cdot 12\ \text{in/ft}}{1.56\ \text{in}}\right)^2} + \sqrt{1 + \frac{0.078 \cdot 0.81\ \text{in}^4 \cdot 1}{68.4\ \text{in}^3 \cdot 17.4\ \text{in}} \cdot \left(\frac{20\ \text{ft} \cdot 12\ \text{in/ft}}{1.56\ \text{in}}\right)^2}$$ [AISC Eq. F2-4]

$$\therefore F_{cr} = 13.59\ \text{ksi}$$

$$M_{ncr} = \frac{F_{cr} \cdot S_x}{12\ \text{in/ft}} = \frac{13.59\ \text{ksi} \cdot 68.4\ \text{in}^3}{12\ \text{in/ft}}$$ [AISC F2.2(c)]

$$\therefore M_{ncr} = 77.49\ \text{kip} - \text{ft}$$

$$M_{nltb} = \min\left(M_{ncr}, M_p\right) = \min\left(77.49\ \text{kip} - \text{ft}, 326.7\ \text{kip} - \text{ft}\right)$$ [AISC Eq. F2-3]

$$\therefore M_{nltb} = 77.49\ \text{kip} - \text{ft}$$

1.5. Controlling Strength

Design flexural strength of the section

$$\phi M_n = \phi_b \cdot \min\left(M_{ny}, M_{nltb}\right) = 0.9 \cdot \min\left(326.7\ \text{kip} - \text{ft}, 77.49\ \text{kip} - \text{ft}\right)$$

$$\therefore \phi M_n = 69.74\ \text{kip} - \text{ft}$$

$$Check\ M_u \leq \phi M_n$$

$$30\ \text{kip} - \text{ft} \leq 69.74\ \text{kip} - \text{ft}$$

$$\therefore OK$$